

Towards bringing Quantum Mechanics and General Relativity together

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Abstract

Two questions are suggested as having priority when trying to bring together Quantum Mechanics and General Relativity. Both questions have a scope which goes well beyond Physics, and in particular Quantum Mechanics and General Relativity.

1. Preliminary Remarks

Bringing Quantum Mechanics and General Relativity together is nowadays considered to be a problem of Physics, and in fact, its fundamental one. Yet not seldom when solving more fundamental problems in any given realm, it may well happen that the sought after solution is less accessible when the ways of thinking are constrained or limited to the respective realm, in this case, the usual ones in Physics.

Let us recall in this regard the celebrated 1960 paper of the Nobel laureate Eugene Wigner, entitled "The unreasonable effectiveness of mathematics in the natural sciences", see [2]. There have been various comments upon, and interpretations of that quite manifest and most impressive "unreasonable effectiveness". And the issue, most likely, is not that Mathematics happens to be more fundamental than Physics, for instance. Rather, it could be about the fact that both Mathemat-

ics and Physics, as they stand today, come from a yet deeper mode of human insight, a mode which, so far, has not been formalized as a science.

In view of such a possibility, it may indeed be useful to try to avoid the exclusiveness of constraining the ways of thinking to the customary ones in Physics, when trying to bring about the unification of Quantum Mechanics and General Relativity.

The ways of thinking in both Mathematics and Physics have their specific qualities and strengths. In Mathematics one is supposed to be perfectly precise, and also most abstract, at least when compared with other natural sciences. Also, creativity in bringing up new ideas and results is rigorously controlled by the requirements of logic, as well as of proofs of conjectures. On the other hand, Physics is significantly more free and protean when it comes to new ideas and hypotheses. Of course, there is again a control, and in fact, a double one this time, namely, of a certain theoretical consistency and of possible supporting experiments, or at least, the lack of contrary ones. The theoretical consistency, however, is not of that extreme rigor as in Mathematics, since much of the more fundamental Physics, among others Quantum Mechanics or Quantum Field Theory, has a somewhat tentative or heuristic mathematical formulation.

This specific quite free and protean nature of thinking in Physics, rather distinct from that in Mathematics among others, brings with it a certain entrapping temptation. Namely, it creates the strong and often irresistible impression, if not in fact of certainly, not only of a generously rich self-contained realm of thinking, but implicitly also of the inappropriateness of any other way of thinking when dealing with problems in Physics.

So much, therefore, for the chance to realize that more fundamental problems in Physics may seriously benefit from ways of thinking which are not constrained or limited to the usual ones in Physics.

And now, let us turn to some of the specifics of the problem of bringing Quantum Mechanics and General Relativity together.

In this regard, it may be instructive to start by recalling the way Special Relativity seems to have emerged in Einstein's thinking. It is often reported that, around the age of 16 or 17, Einstein started to wonder about the following thought-experiment. Assume that a beam of light is emitted from some source S and in the direction A. Einstein then imagined that he himself would move next to that beam of light and do so with the velocity of light. The question which kept puzzled him was : what would he observe ? His answer was that he would only observe some stationary states. For instance, he would keep seeing the source S of light in ever the same state in which it was when it emitted that beam. Further, he would see the light beam next to him as a standing wave in space. And obviously, both of these observations were wrong. After all, the source may change its state with the passing of time, not to mention that standing light waves were not compatible with the Maxwell equations.

The conclusion Einstein drew in 1905 from the above was that the velocity of light could not depend on that of the observer.

This conclusion then became one of the two basic principles of Special Relativity. The other basic principle was the old Galilean one, according to which it is not possible to detect absolute rest or absolute motion.

What happened during the next ten years, while Einstein tried to include gravitation in relativity, may be particularly instructive today, as one attempts to bring together Quantum Mechanics and General Relativity.

The various respective attempts Einstein made over a decade were motivated not so much by what would later be called "general covariance", as rather by a number of specific physical arguments. On the other hand, focusing more on "general covariance" may have helped in simplifying the issues and reaching in a more direct manner the Einstein field equations. After all, there are only two entities invariant under "general covariance", namely, volume and curvature. And the true essence of General Relativity is very much contained in the "general covariance" of the Einstein field equations.

A similar situation was to happen half a century later in Quantum Mechanics with respect to the Bell Inequalities. As it turns out, Bell type

inequalities were known to George Boole in the 1850s, see [1]. Consequently, the Bell Inequalities can be established without any physical arguments. It follows that the original contribution in the Bell inequalities is not in the inequalities themselves, but in showing the fact that they contradict Quantum Mechanics. On the other hand, the clarity of this most simple and fundamental fact is completely lost in the way the Bell inequalities are rather without exception presented and commented upon in a considerable amount of texts written by physicists. Indeed, in all such texts, which limit themselves exclusively to the exhibition of any number of arguments in Physics, the resulting complex buildup of arguments can only serve to obscure the underlying clarity and simplicity of the mentioned fundamental fact.

2. What Is the Local Point of View ?

There appear to be two alternatives when formulating theories of Physics, namely

- (A 1) There can only be a non-local, more precisely, global formulation, which therefore must be unique, or
- (A 2) A variety of local observers can have valid formulations regarding the same physical wholeness which is the object of the theory, just as it is the object in the case of the first alternative (A 1).

So far, throughout the history of Physics, the second alternative (A 2) has been embraced. Moreover, there has been a significant unease, if not even ill-feeling, with respect to physical phenomena which cannot be localized in some suitable manner.

Let us therefore start from the point of view of the second alternative (A 2).

In this case, each local formulation of the physical wholeness is expected to be equivalent in some appropriate way with all the other ones. In other words, we expect a certain principle of "relativity" to hold among the set of such local formulations.

For instance, in Special and General Relativity this is precisely the

case. Furthermore, in these two theories the local aspect corresponds to a given frame of reference, while the equivalence is expressed by the Lorentz, respectively, general covariance.

Turning now to the bringing together of Quantum Mechanics and General Relativity, and doing so along the lines of the second above alternative (A 2), that is, by building local theories, the first question which appears to arise is as follows

- (Q 1) : What is the appropriate local point of view ?

Clearly, the scope of this question is more general than referring only to Quantum Mechanics or General Relativity. In fact, the scope of this question may easily go beyond the whole of Physics as such. Yet its importance, and in fact, priority is quite obvious, in case we start from the point of view of the second alternative (A 2).

Now, coming again back to our problem of bringing Quantum Mechanics and General Relativity together, it is not immediate that the local frames of reference, so essential in both Special and General Relativity, may be sufficient in this case. Also, the non-relativistic classical, or special relativistic frames of reference presently used in Quantum Mechanics may equally be insufficient.

And then, before unleashing the usual protean variety of exclusively physical arguments, it may perhaps be more appropriate to focus on this seemingly more general and deep issue, namely, to try to understand what may an appropriate local point of view be, as asked in (Q 1).

Here, in this regard, it may be useful to recall the following.

In General Relativity there exists a "state space" given by a specific four dimensional Einstein manifold. In non-relativistic Quantum Mechanics of finite systems there exists a "configuration space" given by a suitable Euclidean space, while the "state space" is supposed to be the Hilbert space of square integrable functions defined on the Euclidean space.

This difference alone, not to mention that related to the way the spaces

of "observables" are defined in each of these two theories, may already be reason enough to consider the above question (Q 1).

What has instead happened so far is the following. An exclusive focus was placed on coming up with suitable new "state" or "configuration" spaces. String and Super-string Theory, for instance, postulate as ground physical entities certain continuous geometric structures which take place of the zero dimensional points of classical "state spaces". Alternative approaches, such as in Quantum Gravity, may postulate as an ultimate underlying physical ground various discrete, or locally discrete structures.

Thus none of these approaches sees as a priority answering the above question (Q 1). Consequently, none of these approaches can express in a suitable manner "general covariance", which is the hallmark of the second, that is, local alternative mentioned above in (A 2).

It appears, therefore, that the priority is indeed with answering the above question (Q 1), or in its reformulated manner

- (Q 1*) What is the appropriate concept of "frame of reference" ?

3. Which General Covariance ?

Once question (Q 1), or equivalently, (Q 1*) was answered, one is led to the second question

- (Q 2) What are the requirements of "general covariance" ?

In this regard, it may be quite likely that an answer to question (Q 1) does not necessarily determine, but only limits or conditions the answer to question (Q 2).

4. Conclusions

Needless to say, lots of physical arguments may be involved in answering the above questions (Q 1) and (Q 2).

However, two facts should not be overlooked, namely

- Both questions (Q 1) and (Q 2) have a scope which goes well beyond Physics, and in particular Quantum Mechanics or General Relativity,
- Both questions (Q 1) and (Q 2), in that order, seem to have the highest priority when trying to bring together Quantum Mechanics and General Relativity.

Within alternative (A 2) - which is both the traditional and present day view of Physics - until the emergence of Special and General Relativity it appeared most natural to start first with a view of the physical wholeness which is the object of any given theory in Physics. This wholeness was then modelled, among others, by one or another "state space". Yet ironically, no such "state space" could be defined mathematically, unless some frame of reference was tacitly assumed and employed. Typical in this regard is the absolute space and time of Newtonian Mechanics. In this way, the priority of "frames of reference" was in fact already there, even if implicitly.

Starting with Special Relativity, "frames of reference" obtained an explicit priority in any mathematical model.

A second departure happened with Quantum Mechanics, where in addition to the traditional "state space" given by a suitable Hilbert space, one would now have the "observables" given by self-adjoint operators on that Hilbert space. And strangely enough, the "state space" would now no longer be assumed directly accessible, but only through the "observables".

Such a state of affairs may further support the above point of view regarding the priority in asking questions (Q 1) and (Q 2).

References

- [1] Rosinger E E : George Boole and the Bell inequalities. arXiv: quant-ph/0406004
- [2] Wigner E : The unreasonable effectiveness of mathematics in natural sciences. Comm. Pure Appl. Math., vol. 13, no. 1, February 1960